The graph of $f(x)$ is shown on the right.

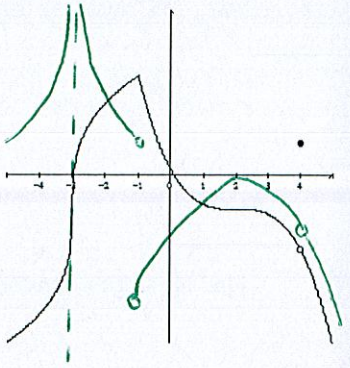
SCORE: _____ / 6 PTS

[a] Find all x-coordinates where $f(x)$ is not differentiable, and explain **very briefly** why.

(1) $x = -3$ VERTICAL TANGENT LINE

(1) $x = -1$ CUSP

(1) $x = 4$ DISCONTINUITY



[b] Sketch a graph of $f'(x)$ on the same axes.

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Find the following limits.
Each answer should be a number, ∞, $-\infty$, or DNE (only if the other answers do not apply).

SCORE: _____ / 7 PTS

$\frac{\infty}{-\infty}$ INDETERMINATE

[a] $\lim_{x\to 1^+} \frac{4-x^2}{1-x^4} = -\infty$ (½) (1)

$\frac{3}{1-1^+} \to \frac{3}{0^-}$

← NO POINTS IF YOUR ONLY ANSWER IS "DNE"

[b] $\lim_{x\to-\infty} \frac{\sqrt{7x^6-5}}{13x^3+11x^2}$

$= \lim_{x\to-\infty} \frac{\sqrt{7x^6-5}}{13x^3+11x^2} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}}$

$= \lim_{x\to-\infty} \frac{\sqrt{7x^6-5}}{13x^3+11x^2} \cdot \frac{-\sqrt{\frac{1}{x^6}}}{\frac{1}{x^3}}$ (½) (1) (½)

$= \lim_{x\to-\infty} -\frac{\sqrt{7-\frac{5}{x^6}}}{13+\frac{11}{x}}$ (1)

$= -\frac{\sqrt{7-0}}{13+0}$

$= -\frac{\sqrt{7}}{13}$ (½) (½)

[c] $\lim_{x\to\infty}(e^{-x} - 6\tan^{-1}x)$

$= \lim_{x\to\infty} e^{-x} - 6\lim_{x\to\infty} \tan^{-1}x$

$= 0 - 6\cdot\frac{\pi}{2}$ (1)

$= -3\pi$ (½)

Let $f(x) = \frac{x}{8-x}$.

SCORE: _____ / 8 PTS

[a] Find $f'(x)$.

(1) $\lim_{h\to 0} \dfrac{\frac{x+h}{8-(x+h)} - \frac{x}{8-x}}{h}$

$= \lim_{h\to 0} \dfrac{(x+h)(8-x) - x(8-x-h)}{h(8-x-h)(8-x)}$

$= \lim_{h\to 0} \dfrac{8x + 8h - x^2 - hx - 8x + x^2 + xh}{h(8-x-h)(8-x)}$ (1 1/2)

$= \lim_{h\to 0} \dfrac{8}{(8-x-h)(8-x)} = \dfrac{8}{(8-x)^2}$ (1) (1)

[b] Find the slope-point form of the equation of the tangent line to the curve of $f(x)$ at the point where $x = 6$.

SLOPE $= f'(6) = \frac{8}{2^2} = 2$ (1)

POINT $= (6, f(6)) = (6, \frac{6}{2}) = (6, 3)$

$y - 3 = 2(x-6)$ (1)

[c] The position of an object moving along a line is $s(t) = \frac{t}{8-t}$ inches, where t is the time in seconds.

Find the instantaneous velocity of the object at time $t = 2$ seconds. Give the correct units for your answer.

$s'(2) = \frac{8}{6^2} = \frac{2}{9}$ INCHES/SECOND (1/2)

(1)

Determine if the function below is continuous at $x = -1$.

SCORE: _____ / 4 PTS

State your conclusion clearly, and show whether each condition in the definition of "continuous (at a point)" is true or false.

In addition, if it is not continuous, determine the type of discontinuity,
and state the conditions in the definition of that type of discontinuity that are true.

$$f(x) = \begin{cases} x^3 - x^4 - x^6, & \text{if } x < -1 \\ -3, & \text{if } x = -1 \\ x^7 + x^5 - x, & \text{if } x > -1 \end{cases}$$

(½) f HAS A JUMP DISCONTINUITY @ $x = -1$

SINCE (1) $\lim_{x \to -1^+} f(x)$ AND $\lim_{x \to -1^-} f(x)$ BOTH EXIST BUT ARE NOT EQUAL

(½) $f(-1) = -3$ EXISTS

(½) $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (x^7 + x^5 - x) = -1 - 1 + 1 = -1$

(½) $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (x^3 - x^4 - x^6) = -1 - 1 - 1 = -3$

$\lim_{x \to -1} f(x)$ DNE (½)

SO, f IS NOT CONTINUOUS AT $x = -1$ (½)

Using complete sentences & proper mathematical notation, **SCORE: _____ / 3 PT**
write the formal definitions of the following terms as shown in lecture.

[a] derivative (function)

THE DERIVATIVE OF f IS $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

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[b] continuous (at a point)

f IS CONTINUOUS AT a IF $f(a)$ EXISTS, $\lim_{x \to a} f(x)$ EXISTS AND $\lim_{x \to a} f(x) = f(a)$

Your score on a test depends on how much time you spent studying for it the day before the test. **SCORE: _____ / 2 PTS**

Suppose $P = f(s)$, where P is your score on the test, in points, and s is the number of hours you studied for it the day before.

What does $f'(3) = 5$ mean ? Your answer must use all the numbers from that equation, and the correct units for those numbers.

NOTE: Your answer must NOT use "slope", "change" nor "derivative".

IF YOU STUDIED 3 HOURS THE DAY BEFORE THE TEST,
YOU WOULD SCORE 5 MORE POINTS FOR EACH ADDITIONAL HOUR
YOU HAD STUDIED

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